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Accuracy Considerations of Power-Ground Plane Models

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Abstract:
Power and ground planes in high-speed printed-circuit board stackups need to be considered as two-dimensional transmission lines. In circuit simulations, the analytical expression of the plane impedance provides an easy means to compute the impedance at any arbitrary location. The expression of impedance contains a double infinite summation of modal harmonics, which in practical calculations must be truncated. This paper discusses the effect of truncation, and it is shown that the summation limits should be set according to the dimensions and loss characteristics of the power-distribution planes.

I. Introduction

With increasing clock speeds and decreasing supply voltages, the wide-band transient current injected into the power and ground planes of printed-circuit boards may create too large noise voltages. To simulate the power distribution system (PDS) with power/ground planes, various models have been proposed. The goal of the simulation is to predict the noise voltage along the PDS with and without bypass capacitors.

In circuit simulations, a widespread plane model is the bedspring matrix of RLGC components or (lossless or lossy) transmission lines [1]. Once the bedspring equivalent circuit is created, its validity is not limited by the circuit simulation. However, the usable bandwidth of bedspring models is limited by some aspects of the processes, which are used to create them:

- Finite granularity of the equivalent circuit. The accuracy of model gradually decreases at frequencies where the delay through one segment of the circuit becomes a non-negligible fraction of the period of signal.
- Truncation of component location. The simple bedspring model has a uniform grid step along one or both axes. If transient sources, bypass capacitors or probe points are located off grid, their locations should be truncated/adjusted to the nearest grid point.

One promising alternative to the bedspring equivalent circuit is the analytical impedance expression of the power/ground planes, which was originally derived for microwave resonators and patch antennas [2]. Not limited by a fixed grid of components, the analytical expression can be solved for any set of arbitrary points on the planes. The expressions yield the self and/or transfer impedances, and the computation may be very efficient [3].
The generic impedance between any two ports on the planes (assuming negligibly small port dimensions) is given by:

$$Z_{ij}(w) = jwm\,h\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{c_{mn}^{\;2}}{w_x w_y\left(k_n^{\;2}-k^2\right)}\cos\left(\frac{2mp\ x_i}{2w_x}\right)\cos\left(\frac{2np\ y_i}{2w_y}\right)\cos\left(\frac{2mp\ x_j}{2w_x}\right)\cos\left(\frac{2np\ y_j}{2w_y}\right)$$

where $\omega=2\pi f$ is the angular frequency
μ is the permeability of dielectric ($\mu=\mu_0=4\pi 10^{-7}$)

$$k_n^{\;2} = \left(\frac{mp}{w_x}\right)^2 + \left(\frac{np}{w_y}\right)^2$$

$$c_{mn} = 1\, for\;\; m = 0\, and\; n = 0;\;\; \sqrt{2}\;\; for\;\; m = 0\, or\; n = 0;\; 2\;\; for\; m \neq 0, n \neq 0$$

c is the speed of light, and for lossless structures

$$k = w\sqrt{em} = w\sqrt{e_r e_0 m_0} = \sqrt{e_r}\,\frac{w}{c}$$

For structures with light losses, the imaginary part of *k* may represent the lossy nature:
k = k' – jk", where *k' = k* above.

Though not limited by finite spatial granularity, the analytical expression has a double infinite series, which for practical calculations must be truncated, so that instead of being infinite, n_max = N, and m_max = M. Truncation of the modal harmonic series will also occur by the gradually increasing high-frequency losses. This paper addresses some aspects of these limitations.

II. Simulation of plane resonances

The plane-impedance expression contains a double series of second-order terms. These terms accurately describe the poles (peaks) in the impedance profile, and the frequencies of the peaks do not change as we add or remove terms. The minima of the impedance profile, however, do change as more terms are added to the series. More importantly, beyond the frequency of the last pole of the truncated series, as opposed to the inductive upslope of the plane impedance at high frequencies, the truncated series yields an impedance of capacitive downslope. Also for this reason, the spreading inductance of planes can be obtained from the expression only with large summation limits.

To show these effects, a pair of 10-inch square planes with 2-mil separation was simulated. Figure 1 shows the simulated self impedance at the corner of planes with different N=M summation limits. While impedance peaks line up for all values of N=M, the frequencies of minima vary. Figures 1/c and 1/b show that the frequency of the first minimum shifts in a range of almost 2:1.

III. Correlation to measurement results

Though not in a step-like function like the truncation of the series, the higher-order modal harmonics are also attenuated by increasing losses. However, for a given set of materials, the cutoff frequency is constant, while the number of modal harmonics below the cutoff frequency depends on the size of planes. Larger planes have lower resonance frequencies and therefore more harmonics should be summed up.

To correlate simulated and measured results, a pair of planes with FR4 dielectrics and 31-mil plane separation was measured at the corner with the two-port self-impedance setup [4]. The 10-inch square planes were then cut in half and quarter, thus increasing the resonance frequencies, while the cutoff frequency of the material was constant. As shown in figures 2/b, 2/c, and 2/d, accuracy of the simulated minimum is maintained only if the summation limit is changed according to the changing cutoff-frequency-resonance-frequency ratio.

IV. Conclusions

The analytical expression of plane impedances is a promising replacement of the bedspring equivalent circuit. The double infinite series of second-order terms can be used to calculate the steady-state self and transfer impedances at any arbitrary set of points. In numerical calculations the infinite series is truncated, which results in a shift of frequencies of impedance minima. To get the correct minimum values, the summation limit must match the cutoff frequency of the structure, which yields a summation limit being dependent of the size of planes.

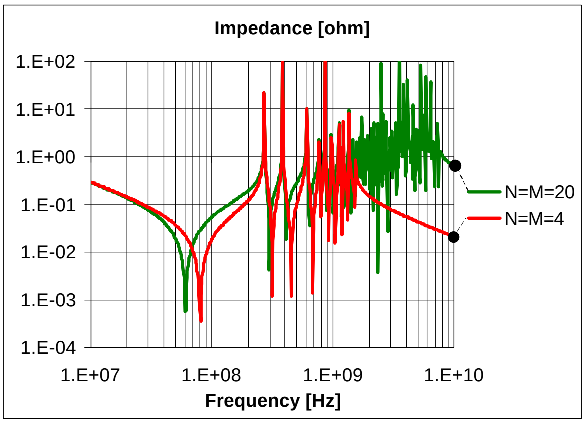

Fig. 1/a

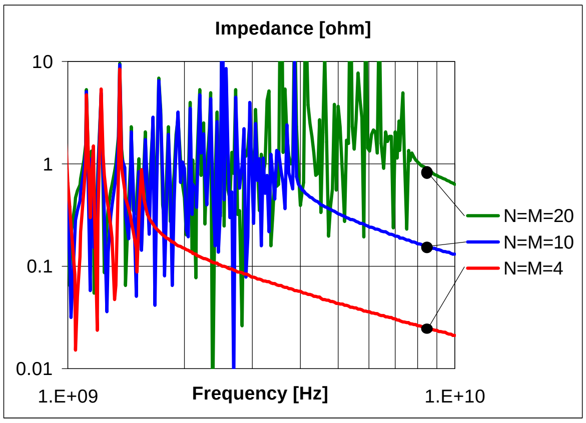

Fig. 1/b

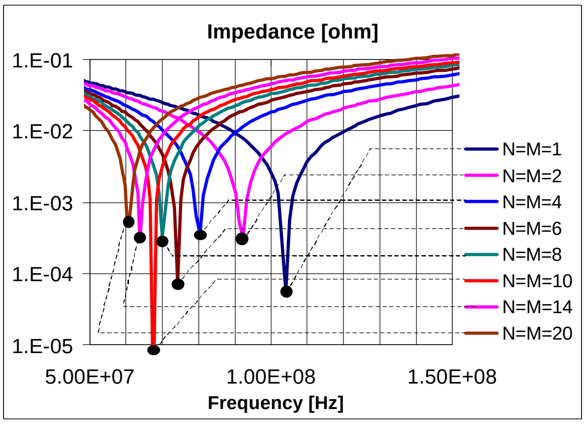

Fig. 1/c

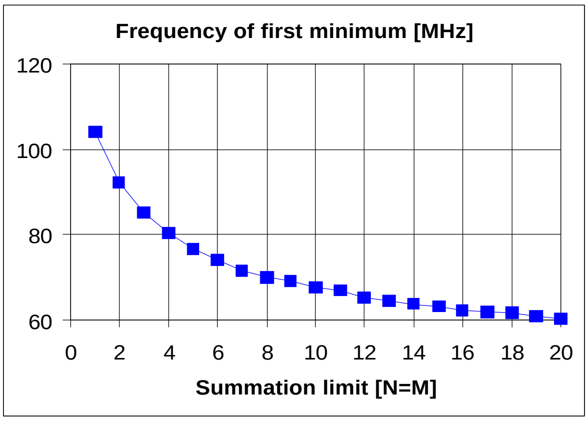

Fig. 1/d

Figure 1. Self impedance of a 10-inch by 10-inch pair of planes, with 2-mil plane separation and FR4 dielectrics. Impedance is simulated by the analytical plane expression, at the corner of planes. Fig. 1/a: Full span of impedance with two different summation limits, N=M=4, and N=M=20. Fig. 1/b: High-frequency impedance of planes with N=M=4, N=M=10, and N=M=20. Fig 1/c: low-frequency impedance of planes near the lowest resonance frequency. Fig. 1/d: Variation of the lowest resonance frequency as a function of summation limit in the range of N=M=1 to N=M=20.

References:

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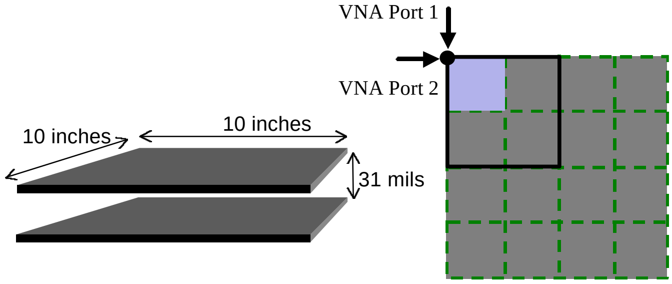

Fig. 2/a

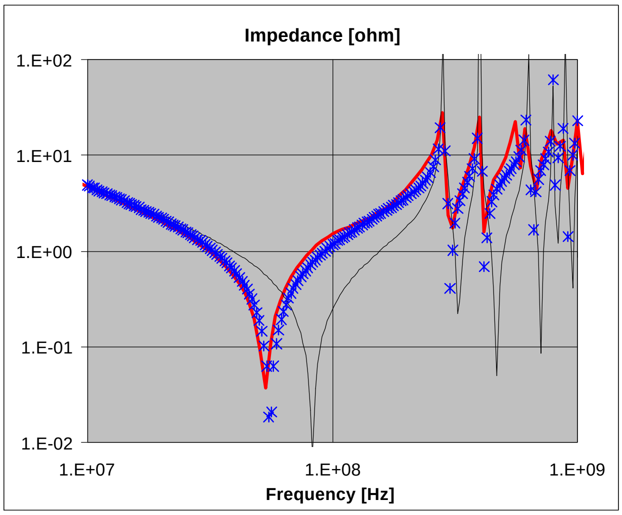

Fig. 2/b

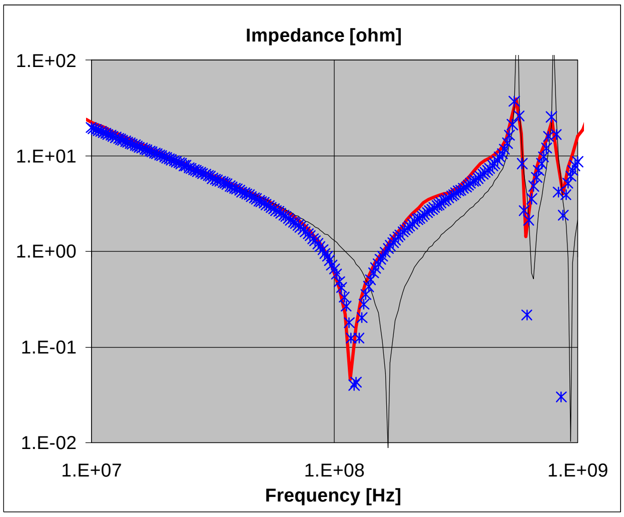

Fig. 2/c

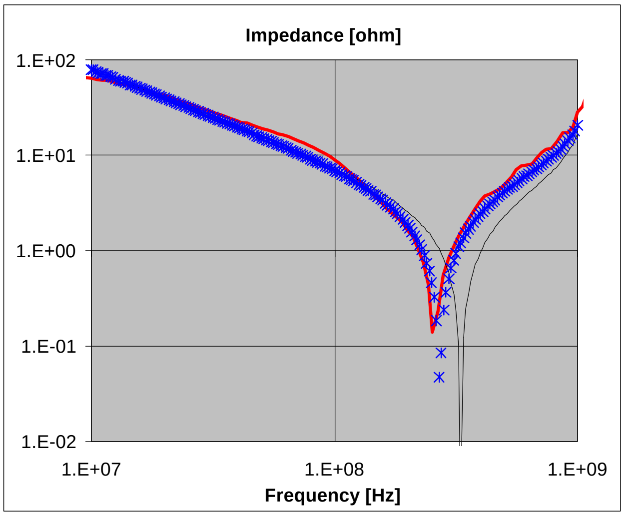

Fig. 2/d

Fig. 2. Simulated and measured self impedance of a pair of planes, with 2-mil plane separation and FR4 dielectrics. Fig. 2/a: Dimensions and connections. The impedance was measured and simulated at the corner. The initial plane size was 10 inch square, which was later cut to 5 inch square and 2.5 inch square. Fig. 2/b: Self impedance of the 10-inch square planes. Heavy line: measured, crosses: simulated with N=M=50, thin line: simulated with N=M=4. Fig. 2/c: Self impedance of the 5-inch square planes. Heavy line: measured, crosses: simulated with N=M=25, thin line: simulated with N=M=4. Fig. 2/d: Self impedance of the 2.5-inch square planes. Heavy line: measured, crosses: simulated with N=M=12, thin line: simulated with N=M=4.