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Correction of Time Domain Measurement Data of Vector Network Analyzers

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Abstract - Vector Network Analyzers (VNA) measure tested circuits in the frequency domain but their frequency range does not include DC. When VNAs are used for time-domain measurements, the proper transformation requires the measured DC value as well. Under certain conditions extrapolation of the DC response can be incorrect, what causes incorrect time domain response, too. This paper presents a solution to this problem.

I. INTRODUCTION

Time-Domain Reflectometry (TDR) and Time-Domain Transmission (TDT) measurements are often necessary to characterize the electrical performance of components and systems. Time-domain measurements can be easily performed in the time domain by connecting a known waveform to the input of the Device Under Test (DUT) and by measuring its response by an oscilloscope. With the advent of more sophisticated instruments with post-processing capability, the measurement of time-domain response through frequency-domain data and Inverse Fourier Transform has become possible (see Fig. 1). For TDR measurements, instead of measuring the sum of reflected and incident waveforms in the time domain, the reflected wave as a function of frequency is measured, while the amplitude of incident wave is constant.

Assuming continuous sampling of measured values, the time-domain impulse response can be expressed as:

$$k(t) = F^{-1}[\Gamma_{in}(\omega)] \tag{1}$$

Time-domain TDR instruments usually perform step-response measurements:

$$h(t) = \int_t k(t)dt \tag{2}$$

The integral can be performed in the time or frequency domain:

$$h(t) = \int_t F^{-1}[\Gamma_{in}(\omega)]dt \tag{3}$$

$$h(t) = F^{-1}\left\{\frac{1}{\omega}\Gamma_{in}(\omega)\right\} \tag{4}$$

The above equivalency is valid only for linear systems; therefore measurement of heavily nonlinear (e.g. active) devices in the frequency domain will generate new problems.

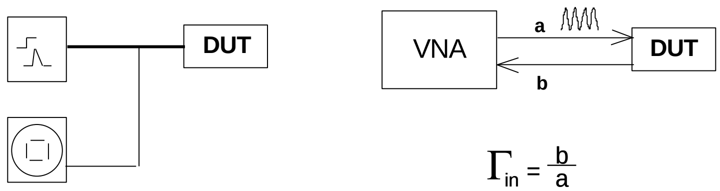

Fig. 1: *Block schematic of TDR measurements in the time and frequency domain.*

Modern instruments work with sampled signals. The measured waveform is digitized at discrete intervals, and one frame (N pieces) of samples is processed. Finite sample number and discretized time or frequency points will result in granularity and periodicity of the data.

$$k(t) = \frac{1}{2N+1} \sum_{n=-N}^{N} \Gamma(2\pi n \Delta f)\, e^{j 2\pi n \Delta f\, t} \tag{5}$$

$$h(t) = \Gamma(\omega = 0)\, t + \sum_{\substack{n=-N \\ n \neq 0}}^{N} \frac{\Gamma(2\pi\, n \Delta f)\, e^{j 2\pi n \Delta f\, t}}{j 2\pi n} \tag{6}$$

With finite-bandwidth VNA measurements, the equivalent time-domain data will also be limited. The rise-time of the equivalent input stimulus is determined by the upper band-limit of VNA. Consequently, the spatial resolution of the measurement is also limited. In some applications the task is to locate discontinuities along transmission lines. For such applications, resolution of the TDR measurements can be increased by suitable postprocessing [3], [4].

The time-span of the equivalent TDR response is determined by the lower frequency limit of the VNA data.

Even though measuring TDR response through frequency-domain data may seem very attractive, a significant drawback of the method stems from the fact that typical vector network analyzers cannot measure the frequency response at zero frequency. In (5) and (6) this means that the data block to be transformed will miss the zero-frequency elements. The missing element would determine the constant term in the *k(t)* function, hence the average slope of the step response. If extrapolation of the DC response from the measured values gives the proper value, the resulting step response is correct. If, however, the extrapolation of DC response is incorrect, the resulting TDR response will also be incorrect, as it is illustrated in Fig. 2. The figure shows the measured TDR impulse and step responses of a 3-meter long coaxial cable. For both measurements, the instrument setting was exactly the same, except the upper frequency limit was 818 MHz, and 820 MHz, respectively.

The instrument selects the measuring frequencies as:

$$stop\_freq \frac{m}{N} \quad m = 1 \ldots N$$

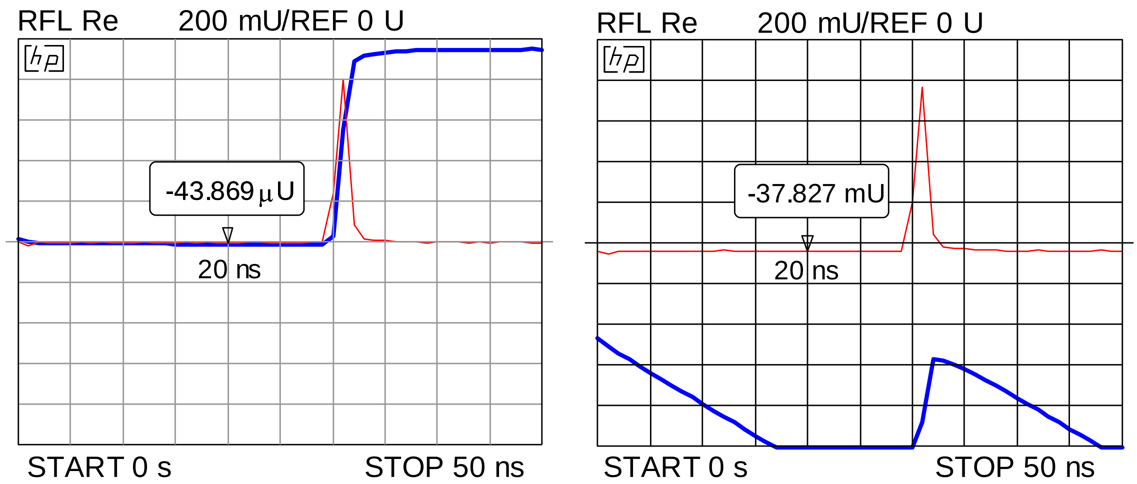

Fig. 2: *Correct and incorrect TDR responses shown by an HP 8752A Vector Network Analyzer. For both cases the DUT was a 3-meter long 50-ohm coax cable with open termination, measured at 51 frequency points. (a, left) Correct TDR response is obtained with a stop frequency of 818 MHz. (b, right) Incorrect TDR response is obtained with a stop frequency of 820 MHz.*

II. DESCRIPTION OF PROBLEM

Let us examine a simple arrangement. If the DUT is a lossless transmission line with open termination at its end, we know that the magnitude of the voltage reflection coefficient at the input is $|\Gamma|=1$. The phase is directly proportional to the frequency. Although the VNA cannot measure at DC, we know that $\Gamma(\omega=0) = +1$ (assuming that the input DC resistance of the DUT is infinite). The instrument must extrapolate this value from the measured lower frequency data. We obtain the correct TDR result only if this extrapolation is successful. As it is shown in Fig. 2, increasing the stop frequency just slightly can significantly deteriorate the step response. What is its reason? Let us have a closer look at the frequency response in Fig. 3. Measurement (b) differs from measurement (a) only in the frequency points, all other settings and the DUT itself was the same. The VNA needs harmonically related frequency-domain data points to generate time domain data, therefore the start frequency must be equal to the stop frequency divided by the number of points. This is why increasing the stop frequency (and leaving the number of points unchanged) will increase the start frequency. The start frequency at the same time equals the frequency increment, Δf. Because of the increased start frequency, in case of (b), the electrical length of the transmission line becomes greater than the quarter wavelength corresponding to the start frequency. Note that in Fig. 3., the input impedance at the start frequency is capacitive for case (a), and inductive for case (b), as shown by the marker readouts on the graphs.

The figure shows the measured frequency-domain data for the same 3-meter long coax in exactly the same setting of VNA that was used to generate Fig. 2.

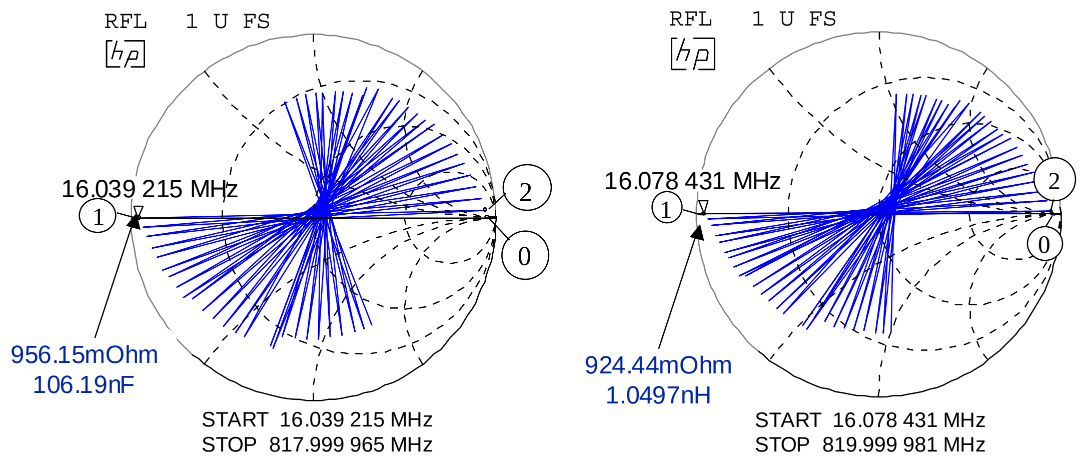

Fig. 3: *Illustration of wrong extrapolation of the zero-frequency data point. (a, left) If the difference between the phases of the zero-frequency element (which is 0 or π) and first measured element is less than π, the extrapolation is correct for $\Gamma(\omega=0)$, see Fig. 2a. (b, right) If the phase difference is greater than π, the extrapolation will fail on the sign of $\Gamma(\omega=0)$, see Fig. 2b. On both graphs 3 points are labeled; 0 denotes the (missing) zero-frequency data, 1 and 2 denote the first and second measured values.*

III. SUGGESTED CORRECTION ALGORITHM

Based on the investigation of several test situations, it was realized that the VNA properly extrapolates the magnitude of the missing zero-frequency response, but under certain conditions fails on the extrapolation of phase.

In the proposed procedure, first the measured voltage-reflection coefficient data is read from the instrument. It is than transformed into the time domain by Inverse Fourier Transform, substituting zero value to the zero-frequency response. In this way we obtain an approximation of the impulse response. The step response can be obtained if before the transformation we divide $\Gamma(n\omega)$ by $(j2\pi n)$ for every $n = 1..N$.

To smooth the time domain result, windowing may be applied either in the frequency or in the time domain. Because we have data only at discrete frequency points, the responses in the time domain will be periodical with a period of $1/\Delta f$. This means that a time domain data at a small negative time τ is equal to the data at the end of the time window minus τ. If this window is wide enough (Δf is small enough) so that the transients of the impulse response die out before the end of the interval, we can determine the DC response so that the impulse response at τ will be zero. If we cannot be sure that the impulse response is sufficiently close to zero at τ, we can calculate the DC response from the frequency domain data. Absolute value and phase also have to be calculated, but it is known that the phase is either 0 or π.

If we could read the extrapolated DC value, it would be simple to produce the corrected impulse and step responses, but only measured data is readable from the VNA. Therefore we have to calculate the time domain response with the DC-response equals zero assumption, at those time points where the instrument does its calculation, and the DC value can be calculated from the difference between the two results. The main steps of the computation are as follows:

Read the step response from the VNA to the vector T_m (windowing must be turned off). Since the number of time domain points is the same as the number of frequency domain points, m runs from 0 to (N-1). The corresponding t_m time points within the block T_m are:

$$t_m = start\_time + m\frac{stop\_time - start\_time}{N-1} \tag{7}$$

Read the complex frequency response data block into the complex vector Γ_n (n = 1..N). Compute the step response:

$$T_m^* = \sum_{n=1}^{N} \mathrm{Im}\left\{\frac{\Gamma_n\, e^{j2\pi n\Delta f\, t_m}}{n\pi}\right\} \tag{8}$$

Now we can calculate the zero-frequency response Γ_0:

$$\Gamma_0 = 6\left|\frac{2\sum_{m=0}^{N-1}\left(T_m^* - T_m\right) - (N-1)\sum_{m=1}^{N-1} m(T_m^* - T_m)}{N(N^2-1)\Delta f\,\Delta t}\right| \tag{9}$$

where Δt is the time step between t_m and t_{m+1}. We have to know a τ time where the step response can be assumed to be zero. Calculate (8) with τ

$$T_{(\tau)}^* = \sum_{n=1}^{N} \mathrm{Im}\left\{\frac{\Gamma_n\, e^{j2\pi n\Delta f\,\tau}}{n\pi}\right\} \tag{10}$$

And now we can tell the *k(t)* impulse response and *h(t)* step response at any t time:

$$k(t) = \frac{\Gamma_0 + 2\sum_{n=1}^{N} \mathrm{Re}\left\{\Gamma_n\, e^{j2\pi n\Delta f\, t}\right\}}{1+2N} \tag{11}$$

$$h(t) = \sum_{n=1}^{N} \mathrm{Im}\left\{\frac{\Gamma_n\, e^{j2\pi n\Delta f\, t}}{n\pi}\right\} - T_{(\tau)}^* + \Delta f\,\Gamma_0\,(t-\tau) \tag{12}$$

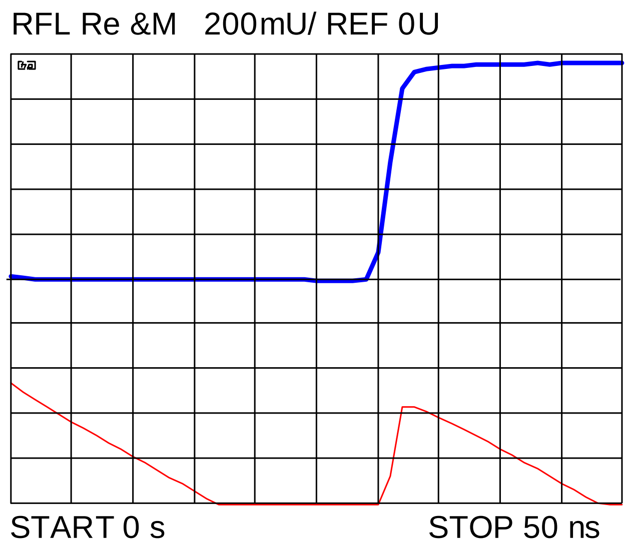

Fig 4: *Incorrect (lower trace) and corrected (upper trace) TDR step response. Compare with Fig. 2.*

IV. MEASUREMENT RESULTS

The corrected and uncorrected impulse responses differ only in a DC shift, which is very small: $\Gamma_0/(2N+1)$. However, the integral of this small DC value can cause a significant slope error in the step response as we see in Fig. 2. It is possible to reload the corrected data into the analyzer and compare with the incorrect one stored in the memory of the instrument. Fig. 4 shows the result of the correction algorithm on the simple test arrangement of Fig. 2.

V. CONCLUSIONS

We have shown how the VNA can be used for time domain measurements. If the lowest measurement frequency is too high, the extrapolated DC response and time domain response will be incorrect. A simple method was introduced to correct this error.

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