# Simulating Planes with SPICE

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There are several excellent commercial tools available for simulating power distribution planes. However, you don't need a commercial tool to do simple plane analysis. You can, for instance, write your SPICE input file and use the free Berkeley SPICE engine to get results.

If you want to do your own plane simulations, there are a couple of simple choices: for rectangular, triangular or circular shapes, you can code the closed-form self- and transfer-impedance expressions in a spreadsheet. One simple implementation for loss-less planes is available at [1]. Losses can also be included in these same calculations if you are willing to handle complex numbers in the spreadsheet. A spreadsheet or any mathematical software package is convenient, because the same application can also plot the results.

Another simple choice to simulate power planes is to create a SPICE netlist and run SPICE. Again, we could use any of the commercially available SPICE tools, most of them having the benefit of offering an integrated schematics capture and graphical post processing user interface. In fact, some of these very good SPICE tools are available for free [2]. On the other hand, using commercial-grade SPICE software, even if it has schematics capture, it is still the user's task to come up with the equivalent circuit for the planes.

One way to create an equivalent circuit for the planes is to map the plane shape to a grid. The bottom sketch of *Figure 1* shows the simplest form, when the solid plane pair has a rectangular contour. For generic plane shapes, with holes and cutouts, and non-rectangular contour, we can use a sufficiently fine or adaptive grid to follow the actual shape.
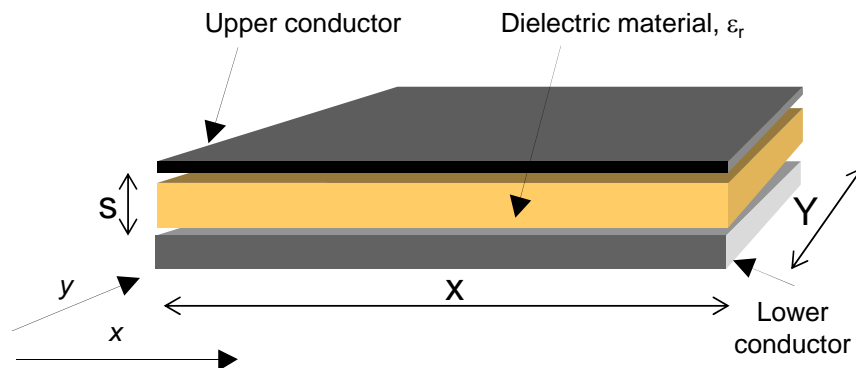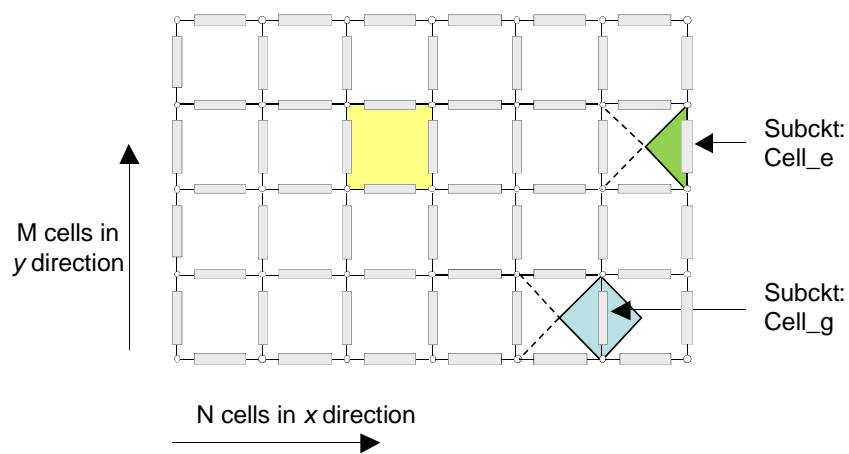
Figure 1: *A pair of rectangular planes with dielectric separation (top) and its SPICE-grid equivalent circuit (bottom).*

We start with overlaying a uniform grid on the rectangular plane shape, dividing the plane pair into square cells (one cell is highlighted by yellow). For this illustration we assume that we can fit precisely N cells along the X length and M cells along the Y width of planes. The cells are represented by four sub-circuits placed along the periphery of each cell. Sub-circuits along the edge of the grid are denoted by Cell_e and represent a triangular shape portion of the plane pair; one of them is highlighted in green. Sub-circuits inside the grid are denoted by Cell_g and represent a square (or diamond) shaped portion of the plane pair; one of them is highlighted in blue. Once we place all of these cells, like tiles, they will cover the entire plane shape continuously, without gap.

The parameters for the Cell_e and Cell_g sub-circuits can be calculated based on the geometry and material properties of the corresponding triangular or diamond shaped plane pair portions they represent. For sake of simplicity here we will use loss-less plane pairs, assuming that the conductivity of the planes is infinite and the dielectric loss tangent of the laminate is zero. Furthermore we assume that the dielectric constant of the laminate is ε_r with a thickness of *s*. We can use two fundamental signal-integrity equations to get the parameters of the sub-circuits: the characteristic impedance and propagation delay of loss-less transmission lines.

$$Z_0 = \sqrt{\frac{L}{C}} \qquad t_{pd} = \sqrt{LC}$$

If we know any two of the four parameters of Z_o, t_{pd}, L and C, we can readily calculate the other two by using these two equations. We start with a cell and calculate its static capacitance (C) and its propagation delay (t_{pd}) from one side to the opposite side.

$$C = \varepsilon_0 \varepsilon_r \frac{\left(X/N\right)^2}{s} \qquad t_{pd} = \frac{\frac{X}{N}}{c}\sqrt{\varepsilon_r} = \frac{X}{N}\sqrt{\varepsilon_r}\sqrt{\varepsilon_0 \mu_0}$$

Here *X/N* is the side dimension of the square cell (based on our conditions, it equals Y/M). Lower-case *c* is the speed of light, which is the inverse square root of two natural constants, ε_0 and μ_0, which are the dielectric constant and permeability of free space, respectively.

With *C* and t_{pd} we can calculate Z_0 and *L*:

$$Z_0 = \frac{t_{pd}}{C} = \sqrt{\frac{\mu_0 s^2}{\varepsilon_0 \varepsilon_r \left(\frac{X}{N}\right)^2}} = 120\pi \frac{s}{X/N} \qquad L = Z_0 t_{pd} = \mu_0 s$$

Note that both the characteristic impedance and the inductance of a square cell depend linearly on the *s* plane spacing; thicker dielectric comes with proportionally higher inductance and impedance. The inductance approximated in this manner (which we may also call the sheet inductance or spreading inductance) does not depend on the size of the square; whereas the characteristic impedance is inversely proportional to it: larger cell comes with lower characteristic impedance.

With the parameters of the yellow cells we can now calculate the parameters of the blue diamonds and green triangles. Since all three have the same distance between its connection end points (which equals the side dimension of a cell), the propagation delay will be the same t_{pd} for all three. Each blue diamond has one half of the area of a cell and therefore its static capacitance will be $C_g = C/2$, its characteristic impedance $2Z_0$ and its inductance $L_g = 2L$. Similarly, each green triangle has one quarter of the area of a cell and therefore its static capacitance will be $C_e = C/4$, its characteristic impedance $4Z_0$ and its inductance $L_e = 4L$. *Figure 3* shows the schematics representation of the sub-circuits.
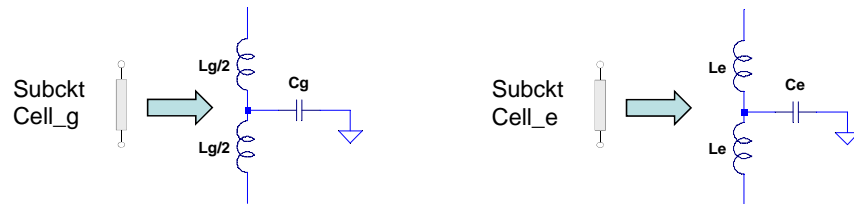

Figure 3: Sub-circuit schematics.

This model uses simple lumped elements, but since it maps the two-dimensional plane pair in both directions, it will correctly capture the modal resonances at all frequencies where the t_{pd} propagation delay of the cells is much less than the period of the highest frequency of interest.

References:
[1]   http://www.electrical-integrity.com/Tool_download_files/Plane_SelfZ_v-w01.xls
[2]   See the listing at http://www.electrical-integrity.com/Links.html under the heading "Free, low-cost, demo and on-line tools"