

Calculating Basic Resonances in the PDN

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In *Figure 2* of the December 2009 column you can see a simple PDN impedance-profile approximation. We showed that the piece-wise-linear Bode plots of the various PDN components can create peaking at some interim frequencies. In general, such kind of peaking occurs when we connect complex impedances in parallel, and at any given frequency one of them is capacitive (with negative phase) while another is inductive (with positive phase). In this column we look at this peaking phenomenon in more detail. We need to talk about this, because unfortunately even today you can find articles, web postings, books and some simple CAD tools giving the wrong answer to this problem.

To start, let us look at the impedance profile of a single bypass capacitor. Any real-life capacitor will have some series resistance and series inductance associated with the capacitor plates and terminals. If we neglect the parallel DC and AC leakage, which is usually an acceptable simplification for PDN applications, we get a series C-R-L equivalent circuit, as shown on the left in *Figure 1*. To understand how to properly calculate the antiresonance between parallel-connected capacitors, a model with frequency-independent C, R and L is sufficient. To emphasize the fact that the antiresonance has a lot to do with the complex nature of the impedances, we plot both the magnitude and the phase of the capacitor's impedance.

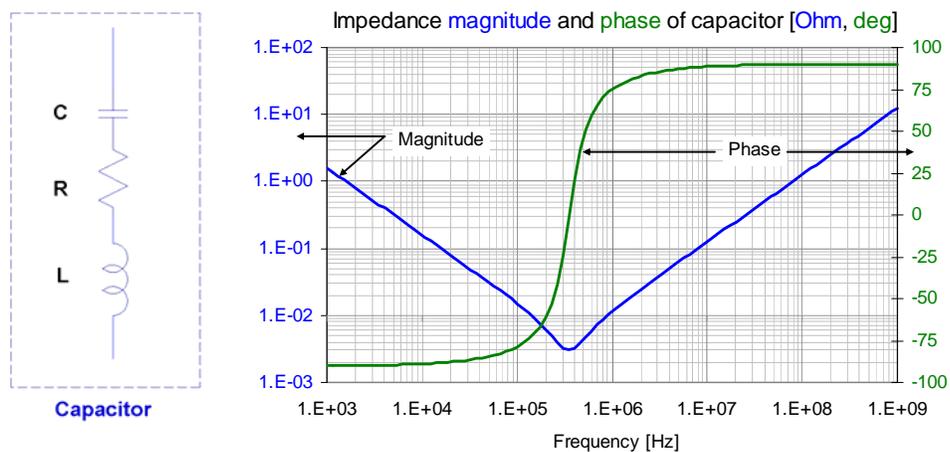


Figure 1: C-R-L equivalent circuit of a bypass capacitor (on the left) and impedance magnitude and phase of a capacitor with $C = 100 \mu F$, $R = 0.003 \text{ Ohm}$, $L = 2 \text{ nH}$ (on the right).

This capacitor has a series resonance frequency (SRF) of 356 kHz. Below SRF the impedance is capacitive: the impedance magnitude slopes downward as frequency

increases and the phase is negative. At SRF the capacitive and inductive reactances cancel, the phase is zero and the impedance magnitude equals R. Above SRF the impedance magnitude slopes upward as frequency increases and the phase is positive.

As a next step we add a second (different) capacitor with $C = 1\mu\text{F}$ $R = 0.01\ \Omega$ $L = 1\text{nH}$ in parallel to the first one. *Figure 2* shows the schematics and the impedances of the two capacitors separately and in parallel. To make sure the figure is not over-crowded, here we show only the magnitudes of impedances.

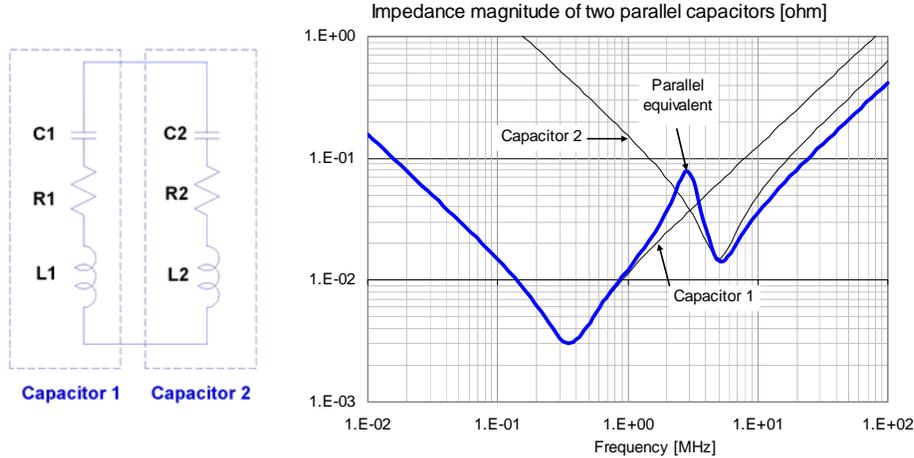


Figure 2: Impedance magnitude of Capacitor 1 (100 μF 0.003 Ω 2 nH) and Capacitor 2 (1 μF 0.015 Ω 1 nH) and their parallel equivalent. The thin lines show the impedance magnitudes of the two individual capacitors; the heavy blue line is the impedance magnitude of their parallel equivalent.

C2 has an SRF of 5.2 MHz. Note that at 3 MHz, between the 356 kHz and 5.2 MHz SRFs of the two capacitors, the thick blue line has a peak with a value of 0.08 Ohms. This peak is much above the values of C1 and C2 impedances at that frequency. This peak, commonly called the antiresonance, is created by the L1 inductance of Capacitor 1 and the C2 capacitance of Capacitor 2.

To properly account for the interactions of the two impedances and for the antiresonance peak, we have to remember that our capacitor equivalent circuits contain series elements and therefore when we connect the two capacitors in parallel, first we have to convert the impedances into admittances. We can then add the complex admittances and convert the sum into impedance. The sequence of calculations:

$$Z_{C1} = R_1 + j\omega L_1 + \frac{1}{j\omega C_1}; Y_{C1} = \frac{1}{Z_{C1}}; Z_{C2} = R_2 + j\omega L_2 + \frac{1}{j\omega C_2}; Y_{C2} = \frac{1}{Z_{C2}}; Z_{sum} = \frac{1}{Y_{C1} + Y_{C2}}$$

The antiresonance peaking is not limited to parallel-connected discrete capacitors: it always happens when at a particular frequency some of the parallel-connected impedances are inductive and some others are capacitive. For instance, we can observe similar peaking when the inductive output impedance of a DC-DC converter resonates with bulk capacitors or when the mounted inductance of bypass capacitors resonates with the static capacitance of power planes. Moreover, for the calculations here we assume lumped connections (zero impedance and delay connecting the capacitors), but similar peaking happens also when the components are distributed over a larger area with finite impedance and delay connecting them.

The proper calculation of the antiresonance involves complex numbers, and if not done properly, we will get erroneous results, which just by looking at the result may appear to be correct. Here we illustrate two common mistakes using the two capacitors from *Figure 2*. The correct response and two typical wrong answers are illustrated in *Figure 3*.

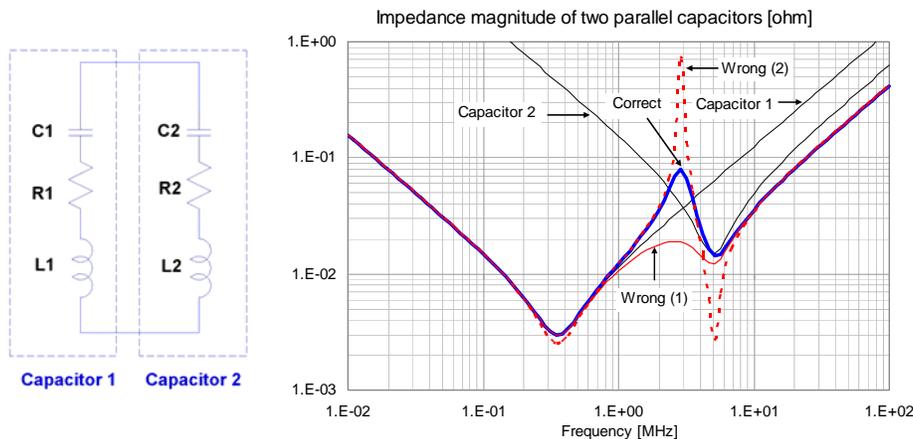


Figure 3: Illustration of two common mistakes when adding capacitors’ impedances in parallel. Correct: blue; wrong: red.

The most typical mistake is to add the two impedance magnitudes in parallel, which is shown by the thin solid red line in *Figure 3*, labeled “Wrong (1)”. Note that the solid red line follows closely the correct result around the minima, but it severely underestimates the antiresonance peak. If we don’t think with the phase of the result, this may look like a correct answer, since in a resistive world we are accustomed to have the sum of parallel-connected resistors being always smaller than any of the two resistor values. I see this mistake happening more often when one of the components is not an actual discrete capacitor. Take for instance the interaction of the output impedance of a DC-DC converter and an external bulk capacitor. We may think that whatever is the output impedance of the DC-DC converter, connecting a capacitor across its output will only lower the impedance. We need to think again: if the converter’s output impedance is inductive at frequencies where the external capacitor’s impedance is still capacitive, in that frequency range the impedance will *increase*.

The second typical mistake is illustrated by the dashed red line in *Figure 3*. It is calculated by summing admittances, but by erroneously calculating the admittances by separately inverting the real and imaginary parts of impedances.

$$\text{correct} : Y_1 = \frac{1}{R_1 + j\omega L_1 + \frac{1}{j\omega C}}; \quad \text{wrong} : Y_1 = \frac{1}{R_1} + \frac{1}{j\omega L_1 + \frac{1}{j\omega C_1}}$$

How big of an error we get due to the incorrect calculations depends on the input numbers. For our case shown in *Figure 2*, the correct antiresonance impedance is 0.08 Ohms. When we erroneously add impedance magnitudes, the peak value is 0.019 Ohms and therefore at that frequency we underestimate the noise by a factor of four. When we erroneously invert the real and imaginary parts of the impedance separately, instead of inverting the complex impedance, the peak value is 0.74 Ohms. In this case we overestimate the noise at that frequency by a factor of nine. This is shown in *Figure 4*, where we take *Figure 3* and we zoom on the horizontal scale between 1 and 10 MHz.

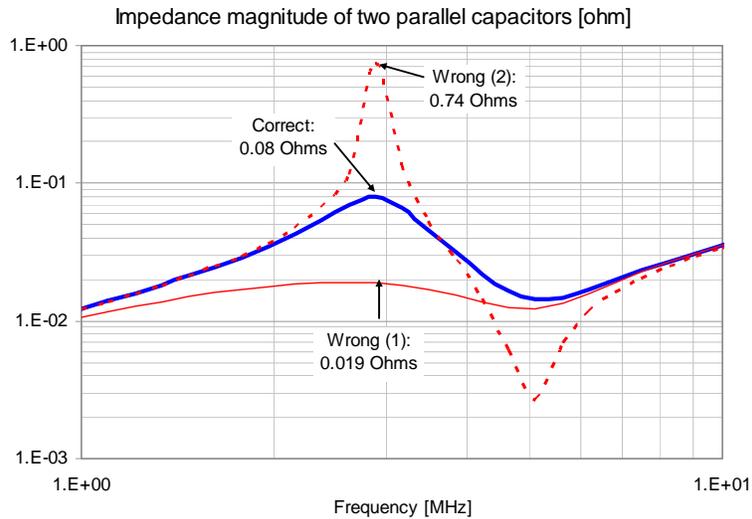


Figure 4: Illustration of the seriousness of calculation errors.

Doing the calculations properly is easy today with CAD tools, math packages and even with spreadsheets. You can download a spreadsheet to calculate the parallel equivalent impedance of three capacitors, including statistical margining, from <http://www.electrical-integrity.com/>.